Calculus I
Chapter 3 Review

Find the derivative with respect to *x* using the limit definition of a derivative.

1. $y = \sqrt{x+3}$

2. $y = \dfrac{x}{x+1}$

Find $\dfrac{dy}{dx}$ for each of the following:

3. $y = 4x^5 + 6x + 9$

4. $y = \dfrac{\sin x + 1}{\sin x - 1}$

5. $y = (2x+1)(3x-5)^2$

6. $y = \sin^2 x \tan 3x$

7. $y = \sqrt{\dfrac{2x-1}{x^2+1}}$

8. $y = \sin(4x^5)$

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9. $y = \left(\dfrac{x}{2-x}\right)^4$

10. $y = \csc(\tan x)$

11. $y = (3x+1)^{-2}(x+5)^4$

12. $y = \cot^2 x$

Find dy/dx using implicit differentiation.

13. $x^2 + 4xy + 5y^2 = x$

14. $5x^2 - xy = y + 2x$

15. Let $y = x^3 - 3x + 4$.

     a. Find the equation of the tangent line at $x = 2$.

     b. For what values of x will the slope of the tangent line equal 0?

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Differentiable functions *f* and *g* have the values shown in the table.

16. If $h(x) = f(x)g(x)$, find $h'(3)$.

x	f	f′	g	g′
0	14	2	3	4
1	9	5	1	6
2	7	6	2	5
3	2	10	0	1

17. If $h(x) = \dfrac{g(x)}{f(x)}$, find $h'(2)$.

18. If $h(x) = f\big(g(x)\big)$, find $h'(1)$.

19. If $h(x) = g^3(x) - f^2(x)$, find $h'(3)$.

20. If $f(x) = x^3 - 4x^2$ on the interval $[-2, 2]$, find the following:

    a. The average rate of change on the interval.

    b. The instantaneous rate of change at $x = 2$.

21. (**Calculator**) Suppose the position function of a moving object is given by $s(t) = \sqrt{t} + \sin t$, where s is in meters and t is in seconds.

    a. Find the average velocity on the interval $[0, \pi]$.

    b. Find the velocity at t = π.

22. (**Calculator**) A baseball is thrown in the air such the height, h, in feet of the ball at any time, t, in seconds is given by the function $h(t) = -16t^2 + 140t + 6$.

    a. Find the average velocity of the ball on the interval [1, 5].

    b. Find the velocity of the ball at $t = 5$. Is the ball going up or down?

    c. At what time will the ball hit the ground?

    d. What is the impact speed of the ball?

    e. At what time will the ball reach its maximum height?

    f. What is the maximum height of the ball?

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Find the derivative with respect to x using the limit definition of a derivative.

1. $y = \sqrt{x+3}$ CONJUGATE

$$\lim_{h \to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}\left(\frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}\right)$$

$$= \lim_{h \to 0} \frac{(x+h+3)-(x+3)}{h(\sqrt{x+h+3}+\sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{x+h+3-x-3}{h(\sqrt{x+h+3}+\sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+3}+\sqrt{x+3}} = \boxed{\frac{1}{2\sqrt{x+3}}}$$

2. $y = \dfrac{x}{x+1}$

$$\lim_{h \to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}}{h} = \frac{x^2+x+xh+h-x^2-xh-x}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{h(x+1-h)}{h(x+h+1)(x+1)} = \lim_{h \to 0} \frac{x+1}{(x+1)^2} = \boxed{\frac{1}{(x+1)^2}}$$

Find $\dfrac{dy}{dx}$ for each of the following:

3. $y = 4x^5 + 6x + 9$

$$y' = \boxed{20x^4 + 6}$$

4. $y = \dfrac{\sin x + 1}{\sin x - 1}$ QUOTIENT RULE

$$y' = \frac{(\sin x - 1)(\cos x) - (\sin x + 1)(\cos x)}{(\sin x - 1)^2}$$

$$= \frac{\sin x \cos x - \cos x - \sin x \cos x - \cos x}{(\sin x - 1)^2} = \boxed{\frac{-2\cos x}{(\sin x - 1)^2}}$$

5. $y = (2x+1)(3x-5)^2$ PRODUCT RULE

$$y' = (2x+1)\cdot 2(3x-5)(3) + (3x-5)^2(2)$$

$$= \boxed{6(2x+1)(3x-5) + 2(3x-5)^2}$$

6. $y = \sin^2 x \tan 3x = (\sin x)^2 \cdot \tan(3x)$ PRODUCT RULE

$$y' = (\sin x)^2 \cdot \sec^2(3x)\cdot 3 + \tan(3x)\cdot 2(\sin x)(\cos x)$$

$$= \boxed{3\sin^2 x \sec^2(3x) + 2\sin x \cos x \tan 3x}$$

7. $y = \sqrt{\dfrac{2x-1}{x^2+1}}$ CHAIN QUOTIENT

$$y' = \frac{1}{2}\left(\frac{2x-1}{x^2+1}\right)^{-1/2}\left(\frac{(x^2+1)(2)-(2x-1)(2x)}{(x^2+1)^2}\right)$$

$$y' = \frac{1}{2}\left(\frac{2x-1}{x^2+1}\right)^{-1/2}\left(\frac{2x^2+2-4x^2+2x}{(x^2+1)^2}\right)$$

$$= \boxed{\frac{1}{2}\left(\frac{2x-1}{x^2+1}\right)^{-1/2}\left(\frac{-2x^2+2x+2}{(x^2+1)^2}\right)}$$

8. $y = \sin(4x^5)$ CHAIN RULE

$$y' = \cos(4x^5)\cdot 20x^4$$

$$= \boxed{20x^4 \cos(4x^5)}$$

9. $y = \left(\dfrac{x}{2-x}\right)^4$

$y' = 4\left(\dfrac{x}{2-x}\right)^3 \left(\dfrac{(2-x)(1) - x(-1)}{(2-x)^2}\right)$

$= 4\left(\dfrac{x}{2-x}\right)^3 \left(\dfrac{2-x+x}{(2-x)^2}\right) = 4\left(\dfrac{x}{2-x}\right)^3 \left(\dfrac{2}{(2-x)^2}\right)$

10. $y = \csc(\tan x)$

$y' = -\csc(\tan x)\cot(\tan x)\cdot \sec^2 x$

PRODUCT

11. $y = (3x+1)^{-2}(x+5)^4$

$y' = (3x+1)^{-2}\cdot 4(x+5)^3 + (x+5)^4 \cdot -2(3x+1)^{-3}(3)$

$= 4(3x+1)^{-2}(x+5)^3 - 6(x+5)^4(3x+1)^{-3}$

12. $y = \cot^2 x = (\cot x)^2$

$y' = 2(\cot x)^1 (-\csc^2 x)$

$= -2\cot x \csc^2 x$

Find dy/dx using implicit differentiation.

13. $x^2 + \overbrace{4xy} + 5y^2 = x$

$2x + 4xy' + y(4) + 10y\cdot y' = 1$

$4xy' + 10yy' = 1 - 2x - 4y$

$y'(4x+10y) = \dfrac{1-2x-4y}{4x+10y}$

14. $5x^2 - xy = y + 2x$

$10x - xy' + y(-1) = y' + 2$

$10x - y - 2 = y' + xy'$

$\dfrac{10x-y-2}{1+x} = y'(1+x)$

15. Let $y = x^3 - 3x + 4$.

    a. Find the equation of the tangent line at $x = 2$.

$f(2) = 2^3 - 3(2) + 4 = 6 \Rightarrow$ POINT $(2,6)$

$f' = 3x^2 - 3 \big|_{x=2} = 9 = m$

$y - 6 = 9(x-2)$

    b. For what values of x will the slope of the tangent line equal 0?

$f' = 0$

$3x^2 - 3 = 0$

$3(x^2-1) = 0$

$3(x-1)(x+1) = 0$

$x = \pm 1$

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Differentiable functions f and g have the values shown in the table.

16. If $h(x) = f(x)g(x)$, find $h'(3)$.

$h' = f(x)g'(x) + g(x)f'(x)$

$h'(3) = f(3)g'(3) + g(3)f'(3) = 2(1) + 0(10) = \boxed{2}$

x	f	f'	g	g'
0	14	2	3	4
1	9	5	1	6
2	7	6	2	5
3	2	10	0	1

17. If $h(x) = \dfrac{g(x)}{f(x)}$, find $h'(2)$.

$h' = \dfrac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2} \Big|_{x=2} = \dfrac{f(2)g'(2) - g(2)f'(2)}{[f(2)]^2} = \dfrac{7(5) - 2(6)}{49} = \boxed{\dfrac{23}{49}}$

18. If $h(x) = f(g(x))$, find $h'(1)$.

$h' = f'(g(x)) \cdot g'(x) \Big|_{x=1} = f'(g(1)) \cdot g'(1) = f'(1) \cdot g'(1) = 5(6) = \boxed{30}$

19. If $h(x) = g^3(x) - f^2(x)$, find $h'(3)$.

$h' = 3g^2(x) \cdot g'(x) - 2f(x) \cdot f'(x) \Big|_{x=3} = 3(0)^2 \cdot (1) - 2(2)(10) = \boxed{-40}$

20. If $f(x) = x^3 - 4x^2$ on the interval $[-2, 2]$, find the following:

   a. The average rate of change on the interval.

   $\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{16}{4} = \boxed{4}$

   b. The instantaneous rate of change at $x = 2$.

   $f' = 3x^2 - 8x$

   $f'(2) = 3(2)^2 - 8(2)$

   $= \boxed{-4}$

21. (**Calculator**) Suppose the position function of a moving object is given by $s(t) = \sqrt{t} + \sin t$, where s is in meters and t is in seconds.

   a. Find the average velocity on the interval $[0, \pi]$.

   $\dfrac{s(\pi) - s(0)}{\pi - 0} = \dfrac{\sqrt{\pi} + \sin \pi - (\sqrt{0} + \sin 0)}{\pi}$

   $= \boxed{\dfrac{\sqrt{\pi}}{\pi}}$

   b. Find the velocity at $t = \pi$.

   $V = s' = \tfrac{1}{2}t^{-1/2} + \cos t$

   $V(\pi) = \dfrac{1}{2\sqrt{\pi}} + \cos \pi$

   $= \boxed{\dfrac{1}{2\sqrt{\pi}} - 1}$

22. (**Calculator**) A baseball is thrown in the air such the height, h, in feet of the ball at any time, t, in seconds is given by the function $h(t) = -16t^2 + 140t + 6$.

a. Find the average velocity of the ball on the interval [1, 5].

$$\frac{h(5) - h(1)}{5 - 1} = \frac{306 - 130}{4} = \boxed{44 \text{ Ft/s}}$$

b. Find the velocity of the ball at $t = 5$. Is the ball going up or down?

$$v(t) = h'(t) = -32t + 140$$

$$v(5) = -32(5) + 140 = \boxed{-20 \text{ ft/s}}$$

c. At what time will the ball hit the ground? $\quad h(t) = 0$

$$-16t^2 + 140t + 6 = 0 \qquad \text{SOLVE w/ CALCULATOR}$$

$$\boxed{t \approx 8.7926}$$

d. What is the impact speed of the ball?

$$v(8.7926) \approx -141.3647 \text{ Ft/s} \qquad \text{SPEED} = |v| = \boxed{141.3647 \text{ Ft/s}}$$

e. At what time will the ball reach its maximum height? $\quad v(t) = 0$

$$-32t + 140 = 0$$

$$t = \frac{140}{32} = \boxed{4.375 \text{ SECONDS}}$$

f. What is the maximum height of the ball?

$$h(4.375) = \boxed{312.25 \text{ Ft}}$$